Published by Institute of Physics Publishing for SISSA/ISAS

Received: *November 28, 2002*
Accepted: *January 19, 2003*

Comment on "Pressure of hot QCD at large N_f"

André Peshier

Institut für Theoretische Physik, Universität Giessen
35392 Giessen, Germany
E-mail: Andre.Peshier@theo.physik.uni-giessen.de

Abstract: It is argued why quasiparticle models can be useful to describe the thermodynamics of hot QCD excluding, however, the case of a large number of flavors, for which exact results have been calculated by Moore.

Keywords: 1/N Expansion, Thermal Field Theory, QCD.

In a recent paper [1], Moore considered the thermodynamics of hot QCD with a large number of quark flavors, $N_f \gg N_c$. Although this case is physically not directly relevant, it is still interesting to study the limit $N_f \to \infty$ as the next-to leading order result in the $1/N_f$ expansion of the pressure can be calculated exactly as a function of the effective coupling $G^2 = N_f g^2/2$ in this model.[1] Therefore, it is suggestive to use the large-N_f limit of QCD as a testing ground for the convergence of perturbative results [2] or resummation improved methods [3, 4, 5]. From the strong-coupling behavior of the pressure at large N_f it was also concluded in [1] that the quasiparticle model [6] cannot be a good description for 'real' QCD near the QCD transition. In the following it is argued why such a conclusion has to be taken with some care.

To start with, a sketchy derivation of the large-N_f pressure is given within the Φ-derivable approach. Setting the volume of the system to one, the QCD pressure for arbitrary N_f can be expressed in terms of the full quark and gluon propagators [4, 5] (the ghost contribution and the subtraction of the vacuum pressure are not shown explicitly),

$$p = \mathrm{Tr}\left[\ln\left(-S^{-1}\right) + \Sigma S\right] - \frac{1}{2}\mathrm{Tr}\left[\ln\left(-D^{-1}\right) + \Pi D\right] + \Phi[S, D]\,. \tag{1}$$

The traces are taken over the 4-momentum and, accordingly, over the flavor, spin, color and Lorentz indices. Φ is the sum of all 2-particle irreducible bubble diagrams. The self-energies are obtained by cutting a corresponding line in Φ. In the large-N_f limit, Φ reduces to the 1-gluon exchange diagram for each flavor and contributes $\frac{1}{2}\mathrm{Tr}\Pi D$ to p, which compensates the second term in the gluon contribution. While the gluon self-energy is of the order G^2, the quark self-energy is of the order G^2/N_f. Thus, the leading term in the $1/N_f$ expansion of p is given by the free quark pressure, $p_{lo} = \mathrm{Tr}\ln(-S_0^{-1}) \sim N_f$. The term of the order N_f^0 comes from the remaining gluon contribution [1],

$$p_{\mathrm{nlo}} = -\frac{1}{2}\mathrm{Tr}\ln\left(-D^{-1}\right). \tag{2}$$

Moore's numerical computation for massless quarks shows that p_{nlo} first decreases with increasing G^2 as expected, but then starts to increase again, exceeding even the free value $p_{\mathrm{nlo}}(G^2 = 0)$ before the coupling becomes eventually so large that the position of the Landau pole reaches the order of the temperature and the model becomes meaningless.[2] Clearly, such a non-monotonic behavior cannot be described by quasiparticles whose masses rise with the coupling.

To understand the surprising behavior of the large-N_f pressure, and whether it might be relevant for real QCD, it is instructive to go back to an arbitrary number of flavors.

[1]Note that the resulting theory is not asymptotically free and leads to a Landau pole, so it is not defined in a strict sense. However, as argued in [1], the model can be understood as an effective theory with a controllable ambiguity which is small as long as the Landau pole is much larger than any other scale in the problem.

[2]After submitting this article, Moore's numerical results for p_{nlo} as published in [1] were corrected in [7]. While the pressure still has a minimum, it now exceeds the free pressure only at a value of the coupling where it is already sensitive to the Landau pole ambiguity.

	$N_f = 0$	$N_f = 3$	$N_f = 6$	large N_f
g_2	3.2	3.7	3.9	$G_2 = 3.5$
g_3	0.79	0.96	0.97	$G_3 = 3.4$
δ	0.0087	0.0099	0.0094	0.14

Table 1: The values of the coupling indicating the validity range of the quadratic approximation $p^{(2)}$ of the pressure for several N_f and of $p_{\rm nlo}$ in the large-N_f limit, respectively, and the depth δ of the minimum for the cubic approximation.

Then, in dimensionless units and up to terms of order g^4, the perturbative expansion of the pressure reads [2]

$$p^{(3)}(N_c, N_f) = \frac{7}{4}\frac{d_f}{d_a} + 1 - 5\left(N_c + \frac{5}{4}N_f\right)\bar{g}^2 + \frac{80}{\sqrt{3}}\left(N_c + \frac{1}{2}N_f\right)^{3/2}\bar{g}^3\,, \tag{3}$$

with $d_f = N_c N_f$, $d_a = N_c^2 - 1$, and $\bar{g} = g/(4\pi)$. The normalization is such that the free gluon contribution is one. Note that to this order the result does not depend explicitly on the renormalization scale. Aside from the fact that in general the terms beyond $\mathcal{O}(g^5)$ in the expansion of p cannot be calculated by perturbative methods, the series is not convergent. The best one can hope for is some sort of an asymptotic expansion. Such a series, however, should be truncated at a certain order, depending on the expansion parameter, to give the best approximation possible. Typically, one should truncate the series at the nth term when its modulus becomes as large as that of the preceding contribution, see, e. g., [8]. Applying this prescription to the expansion (3) leads to the estimate that $p^{(3)}$ is a reasonable approximation for $\bar{g}$ smaller than

$$\bar{g}_3 = \frac{\sqrt{3}}{16}\,\frac{N_c + \frac{5}{4}N_f}{(N_c + \frac{1}{2}N_f)^{3/2}}\,. \tag{4}$$

$p^{(3)}$ has a minimum at $\bar{g}^\star = \frac{2}{3}\bar{g}_3$ (which therefore might be physically relevant) with a relative depth of

$$\delta = 1 - \frac{p^{(3)}(\bar{g}^\star)}{p^3(0)} = \frac{5}{144(4 + 7d_f/d_a)}\left(\frac{N_c + \frac{5}{4}N_f}{N_c + \frac{1}{2}N_f}\right)^3\,. \tag{5}$$

For $\bar{g} > \bar{g}_3$, the $\mathcal{O}(g^2)$ approximation, $p^{(2)}$, is expected to be more adequate until it obviously also becomes meaningless at

$$\bar{g}_2 = \left(\frac{1}{5}\,\frac{4 + 7d_f/d_a}{4N_c + 5N_f}\right)^{1/2}\,. \tag{6}$$

The values of g_2, g_3 and δ for $N_c = 3$ and several numbers of N_f are given in table 1, and the representative case $N_f = 0$ is illustrated in figure 1.

Due to the estimated large validity range of the approximation $p^{(2)}$ one can expect that the monotonically decreasing behavior at large coupling is qualitatively correct for the exact pressure. This is indeed what is observed in QCD lattice calculations with a small number

JHEP01(2003)040

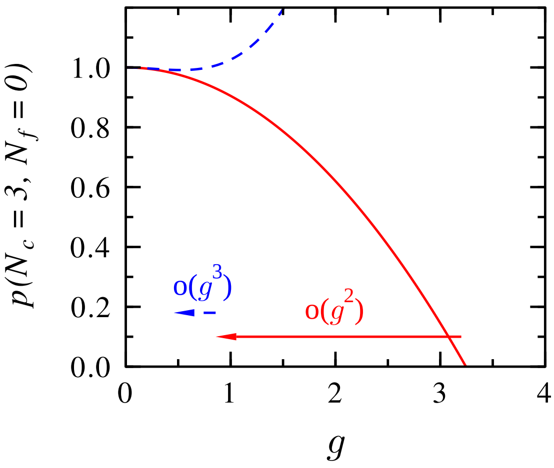

Figure 1: The perturbative approximations, $p^{(2)}$ and $p^{(3)}$, of the pressure for $N_f = 0$, in units of the free value.

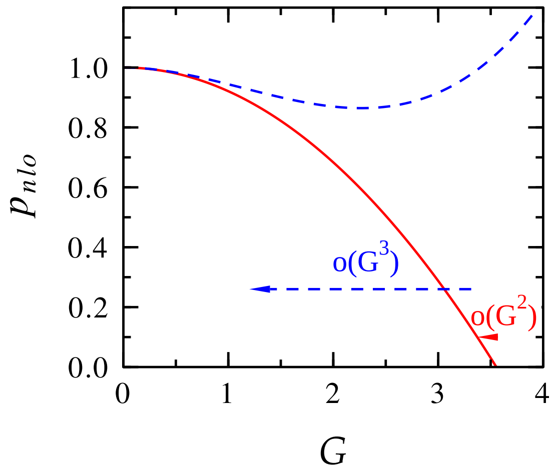

Figure 2: The approximations $p^{(2)}_{\text{nlo}}$ and $p^{(3)}_{\text{nlo}}$ of the large-N_f pressure p_{nlo}, scaled by the free value.

of flavors [9], where the pressure decreases monotonically with the temperature and reaches a rather small value at the transition temperature T_c. Although it is not relevant for the large coupling which is of primary interest here, it is mentioned that the minimum of the approximation $p^{(3)}$ is very shallow, and that the $\mathcal{O}(g^4)$ contribution to p is for reasonable choices of the renormalization scale always smaller than the $\mathcal{O}(g^3)$ contribution.[3] The latter observation might indicate that the perturbative series is not an asymptotic expansion in the strict sense.

Considering now the large-N_f limit, one can read off the perturbative expansion of p_{nlo} from eq. (3) after subtracting the term $\sim N_f$; with $\bar{G} = G/(4\pi)$,

$$p^{(3)}_{\text{nlo}} = 1 - \frac{25}{2}\,\bar{G}^2 + \frac{80}{\sqrt{3}}\,\bar{G}^3\,. \tag{7}$$

The perturbative structure of p_{nlo} is rather different from the case discussed above. Although the $\mathcal{O}(G^2)$ approximation vanishes at a similar value of the (effective) coupling, the estimated (as before) validity range of $p^{(2)}_{\text{nlo}}$ is tiny, see table 1 and figure 2. Therefore, one cannot expect p_{nlo} to become small at large coupling — contrary to the pressure at small N_f. Again, this expectation is in line with the exact result [1]. Numerically, the expression $p^{(3)}_{\text{nlo}}$ turns out to be a reasonable approximation[4] for values of G as large as the coupling at the minimum of p_{nlo}, while $p^{(2)}_{\text{nlo}}$ never seems to be the optimal approximation.

The truncation prescription for asymptotic series implies that the terms of a higher than a certain order, which depends on the size of the expansion parameter, almost cancel each other. For the pressure of QCD with a few flavors, in the region of physical interest, near the QCD transition, this occurs already after the leading order correction in g^2. Hence,

[3] In figure 1, the lower bound of the validity interval of $p^{(3)}$ is estimated from the magnitude of the 4th and 5th order contribution to p.

[4] It is noted that the $\mathcal{O}(G^3)$ contribution is (for relevant values of the coupling) always larger than the $\mathcal{O}(G^4)$ contribution which is larger than the contribution of the order G^5.

JHEP01(2003)040

a quasiparticle model like [6], which incorporates the leading perturbative correction and resums higher order terms only partly to ensure thermodynamic consistency, appears to be justified. The situation is different for the large-N_f limit of QCD where the pressure $p_{\rm nlo}$ has a peculiar perturbative structure. This fact is physically plausible: Here the cubic plasmon term, i. e., the correction to the leading exchange contribution due to screening, is small since quarks screen less than gluons. Consequently, this term is relevant at much larger coupling than in the physical case.

In conclusion, Moore's argument — the quasiparticle model [6] cannot describe the minimum of $p_{\rm nlo}$ at large N_f and is thus not expected to be a good description of the physical case either — would only be convincing if such a pronounced minimum also occurred in real QCD; otherwise the large-N_f limit is governed by different physics. By the argument that in both cases the strong-coupling behavior of the pressure is reflected in the appropriately interpreted perturbative results, or directly by contrasting the numerical results (lattice data for real QCD), this, however, seems indeed to be the case.

Acknowledgments

I thank S. Leupold for comments on the manuscript. This work is supported by BMBF.

References

[1] G.D. Moore, *Pressure of hot QCD at large-N_f*, *J. High Energy Phys.* **10** (2002) 055 [hep-ph/0209190].

[2] C.-x. Zhai and B. Kastening, *The free energy of hot gauge theories with fermions through g^5*, *Phys. Rev.* **D 52** (1995) 7232 [hep-ph/9507380].

[3] J.O. Andersen, E. Braaten and M. Strickland, *Hard-thermal-loop resummation of the thermodynamics of a hot gluon plasma*, *Phys. Rev.* **D 61** (2000) 014017 [hep-ph/9905337]; J.O. Andersen, E. Braaten, E. Petitgirard and M. Strickland, *Htl perturbation theory to two loops*, *Phys. Rev.* **D 66** (2002) 085016 [hep-ph/0205085].

[4] J.P. Blaizot, E. Iancu and A. Rebhan, *Approximately self-consistent resummations for the thermodynamics of the quark-gluon plasma, I. Entropy and density*, *Phys. Rev.* **D 63** (2001) 065003 [hep-ph/0005003].

[5] A. Peshier, *HTL resummation of the thermodynamic potential*, *Phys. Rev.* **D 63** (2001) 105004 [hep-ph/0011250].

[6] A. Peshier, B. Kämpfer, O.P. Pavlenko and G. Soff, *A massive quasiparticle model of the* SU(3) *gluon plasma*, *Phys. Rev.* **D 54** (1996) 2399.

[7] A. Ipp, G.D. Moore, A. Rebhan, *Comment on "Pressure of hot QCD at large-N_f with corrected exact results"*, hep-ph/0301057.

[8] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series and products*, Academic Press San Diego, San Diego 2000.

[9] F. Karsch, E. Laermann and A. Peikert, *The pressure in 2, 2+1 and 3 flavour QCD*, *Phys. Lett.* **B 478** (2000) 447 [hep-lat/0002003].